HOMEWORK 12 - ANSWERS TO (MOST) PROBLEMS

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Section 5.3: The Fundamental Theorem of Calculus

5.3.4.
- (a) $g(-3) = 0$, $g(3) = 0$
- (b) $g(-2) \approx 2$, $g(-1) \approx 4$, $g(0) \approx 6$
- (c) $(-3, 0)$
- (d) 0
- (f) $g'(x) = f(x)$

5.3.7. $\frac{1}{x^3+1}$

5.3.15. $\sec^2(x)\sqrt{\tan(x) + \sqrt{\tan(x)}}$

5.3.17. $3\frac{(1-3x)^3}{1+(1-3x)^2}$

5.3.25. $\frac{7}{8}$ (antiderivative is $-\frac{1}{t^3}$)

5.3.31. 1 (antiderivative is $\tan(t)$)

5.3.35. $\frac{\ln(9)}{2} = \ln(3)$ (antiderivative is $\ln(|x|)$)

5.3.41. $1 + (-1) = 0$ (split up the integral into $\int_0^{\frac{\pi}{2}} \sin(x)dx + \int_{\frac{\pi}{2}}^{\pi} \cos(x)dx$)

5.3.43. $\frac{1}{x^4}$ is discontinuous at 0 (the FTC applies only to continuous functions)

5.3.54. $g'(x) = 2x\frac{1}{\sqrt{2+x^8}} - \sec^2(x)\frac{1}{\sqrt{2+\sec^8(x)}}$

Section 5.4: Indefinite Integrals and the Net Change Theorem

5.4.10. $\frac{v^4}{6} + \frac{2}{3}v^2 + \frac{2}{3} + C$

5.4.12. $\frac{x^3}{3} + x + \tan^{-1}(x) + C$

5.4.13. $-\cos(x) + \cosh(x) + C$

5.4.25. 52 (antiderivative is $3x^3 + 3x^2 + x + \frac{1}{9}$)

5.4.37. $1 + \frac{\pi}{4}$ (antiderivative is $x + \tan(x)$)

5.4.47. $\frac{4}{3}$ (antiderivative is $y^2 - \frac{y^3}{3}$)

5.4.52. The bee population after 15 weeks

Date: Monday, April 25th, 2011.

5.4.58.

(a) $s(3) - s(5) = -\frac{10}{3}$ (antiderivative is $\frac{t^3}{3} - t^2 - 8t$)

(b) $(s(1) - s(4)) + (s(6) - s(4)) = 18 + \frac{44}{3} = \frac{98}{3}$

5.4.59.

(a) $v(t) = t^2 + 4t + 5$

(b) $s(10) - s(0) = \frac{1750}{3}$ (antiderivative is $\frac{t^3}{3} + 2t^2 + 5t$)

5.4.61. $\frac{140}{3}$ (antiderivative is $9x + \frac{4}{3}x^{\frac{3}{2}}$, and $a = 0$, $b = 4$)

5.4.62. 1800 (antiderivative is $200t - 2t^2$, $a = 0$, $b = 10$)

Section 5.5: The substitution rule

5.5.7. $\frac{1}{2}\cos(x^2)$ ($u = x^2$, $du = 2xdx$)

5.5.31. $-\frac{1}{\sin(x)}$ ($u = \sin(x)$, $du = \cos(x)dx$)

5.5.39. $\frac{1}{3}\sec^3(x)$ ($u = \sec(x)$, $du = \sec(x)\tan(x)$)

5.5.46. $\frac{1}{5}(x^2+1)^{\frac{5}{2}} - \frac{1}{3}(x^2+1)^{\frac{3}{2}}$ ($u = x^2 + 1$, $du = 2xdx$, $x^2 = u - 1$)

5.5.59. $e - \sqrt{e}$ ($u = \frac{1}{x}$, $du = -\frac{1}{x^2}dx$, $a = 1$, $b = \frac{1}{2}$)

5.5.73. $0 + 6\pi$ (the first integral is 0 because the function is an odd function, or use $u = 4 - x^2$, $du = -2xdx$, $a = 0$, $b = 0$, and the second integral represents the area of a semicircle with radius 2)

5.5.88.

(a) For the first integral, let $u = \cos(x)$, then $du = -\sin(x)dx = -\sqrt{1-u^2}dx$, so the first integral becomes $\int_1^0 \frac{f(u)}{-\sqrt{1-u^2}}du = \int_0^1 \frac{f(u)}{\sqrt{1-u^2}}du$. For the second integral, let $u = \sin(x)$, then $du = \cos(x)dx = \sqrt{1-u^2}dx$, so the second integral becomes $\int_0^1 \frac{f(u)}{\sqrt{1-u^2}}du$, and it is now clear that both integrals are equal!

(b) By (a) with $f(x) = x^2$ (for the first step), and the fact that $\sin^2(x) = 1 - \cos^2(x)$, we get:

$$\int_0^{\frac{\pi}{2}} \cos^2(x)dx = \int_0^{\frac{\pi}{2}} \sin^2(x)dx = \int_0^{\frac{\pi}{2}} 1dx - \int_0^{\frac{\pi}{2}} \cos^2(x)dx = \frac{\pi}{2} - \int_0^{\frac{\pi}{2}} \cos^2(x)dx$$

Solving for $\int_0^2 \cos^2(x)dx$, we get: $\boxed{\int_0^2 \cos^2(x)dx = \frac{\pi}{4}}$, and hence $\boxed{\int_0^2 \sin^2(x)dx = \frac{\pi}{4}}$ (by (a))